A MATHEMATICAL MODEL FOR LEAK LOCATION AND LEAK AREA DETERMINATION IN PIPELINE NETWORKS

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Prompt leak location and leak area determination in oil and gas pipeline installations is an indispensable approach to controlling petroleum products wastages in pipes. However, there is an evident lack of literature information on this subject. In this paper, we modelled leak location detection and leak area determination in pipes by applying two methodologies and gave an illustrative example using simulated data with the aid of Matlab. A comparison of these two approaches resulted in an error of 6.24%, suggesting that the closer the leak is to the measurement station, the lower will be the time interval between two successive waves that will pass through the leak and get to the measurement station. The relationship between the pipe area and coefficient of reflection is parabolic. This contribution is valuable to pipeline engineers in the economic control of leaks.

KEY WORDS: pressure transducer, waves, detection, measurement, simulation

INTRODUCTION

Nowadays, pipeline transportation plays a prominent role in the economy of the industries – factories in the petrochemical, agro-food, chemical, power generation, and mining. All they prefer pipelines to road transportation of fluids as it is a more efficient way of transporting fluid in a high-flow capacity with minimum or low-cost in the long-term (1). However, pipeline leakages, which are the main faults of pipelines (2), happen due to the factors such as ageing, erosion, abrasion and natural disasters (3) but locating such leakages is very challenging in the complex networks of pipelines. To deal with this problem, which usually involves enormous manpower costs due to staff deployment, search equipment cost and emergency rescue team expenditure in leak location endeavours, scientific methods are needed. The need to measure, monitor, control, and more importantly, secure and direct the flow of flammable and explosive solid powders, liquids and gases that are transported in pipelines is a problem that is generally recognised by the industry, governments, monitoring agencies, and researchers (3, 4). Without an appropriate and prompt leak location and fixing, serious consequences relating to safety, pollution, economy and considerable loss of lives and properties may occur (3).
In the context of this discussion, the principles used in the practice may be helpful in ensuring that the companies comply with laid down policies and standards concerning the safety and environmental regulations in relation to leak detection and location. The drive for compliance through proactive responses in the efforts concerning leak location is visible through the large number of seminars and financial support to universities by the industry aimed at finding out best solution techniques that are efficient, modern, and cost effective. Despite this, there are still many gaps and there exists many companies in developing countries that have serious limitations in developing and utilizing available methods. First, the pipeline engineer responsible for pipeline management activities would have to devote enormous time to look for leakage location solutions, ensure their implementations and give feedback to the suppliers of solutions on a regular basis and in a limited time span. Costs could be another reason since the effective implementation of location solutions requires multiple-point systems and set-ups within the networks, requiring huge amount of manpower, training, safety, and surveillance costs, among others. Thus, there is a great need for cost effective models and techniques on leak location. The risk analysis of leakages in pipelines and detection techniques are well studied in the literature but not the problem of their location. Moreover, during the leakages, the extent of wreckage is determined by the promptness in locating the actual spots and location of leaks, since detection of a leak would only indicate the problem and not specify the leak location needed for immediate corrective actions. Traditionally, leak acoustics and leak detection equipment are used to detect leaks. Listening the rods, leak noise correlators, and aqua phones have been used to locate leaks. However, the costs in terms of multiple points within the network that such equipment need to be placed, has been a major limiting factor to purchase such equipment. Thus, alternatives should be provided. The objective of this paper is to develop a mathematical model for leak location in oil and gas pipeline using pressure transducers.

The following is a brief literature review on the current subject. Pollack (5), Miller et al. (6) and Jiao et al. (7) have all used acoustic emission technique in proposing model for leak detection and location in pipelines installations. The application of frequency response method in detecting leaks in pipelines involves solving differential equations at different domain in the pipelines mode with respect to time; thus method have been extensively used in literature (8, 9, 10). On the other hand, infrared inverse transient analysis (11, 12, 13) and infrared optical imaging technique (14, 15, 16) have gained wide acceptability in detecting and locating leaks in pipeline networks. However, the systems considered in this paper are expensive, requiring high technical know-how. Apart for small oil and gas installation systems, the demand of maintaining some of the models is challenging. The need to develop a system, which serves as an alternative, easy-to-implement with less expertise requirements should be given more attention in view of the current economic challenges world-wide. Hence, the actualization of this need is pursued in this work.

A further contribution to solving the problem considered has been the work of Miller et al. (6), who detected and located leaks of a magnitude of 0.1 m³h⁻¹. Lay-Ekaakille et al. (17) examined leak in a zigzag pipeline using a filter diagonalization method-based algorithm in detecting leaks. Also, Qu et al. (18) designed a pre-warning system for detecting leaks. In all these studies (6, 17, 18) the authors (18) did not use pressure transducers in
detecting leaks. The contributions by Tylman (19) and Mounce et al. (20) utilised artificial intelligence to study the problem in question. The current paper has an edge over traditional methods of leak location in that none of these studies by Tylman (19) and Yang et al. (3) considered the use of pressure valves in locating and detecting leaks, which is an effective and easy-to-apply method. Other authors who have worked in the domain of leak location and detection Mounce et al. (20), Leclitenbohmer et al. (21) and Colombo et al. (1). Yang et al. (3) described a new approach for leak detection that considered velocity of natural gas. Colombo et al. (1) identified direct transient analysis, frequency domain techniques, inverse-transient analysis and transient leak detection as common detection methods. Similarly, Zeidouni et al. (22), de Silva et al. (2), and Lee et al. (23) developed models for leaks detection. In Zeidouni et al. (22), the use of pressure measurement in the presence of noise in obtaining leakage pathway transmissibility and location was described. It was observed that the chi-square function for the same noise distribution for all measurement was reduced to ordinary least-squares.

Furthermore, de Silva et al. (2) used clustering, classification tools and fuzzy system for leak detection operation. They obtained 93.67% and 98.03% correctness for flow and detection evaluations for a LPG pipeline when results from the fuzzy system and specialist conclusions were compared: Lee et al. (23) developed a model that made use of frequency response diagram in determining the actual location size of leak within a pipeline. The model is not appropriate for detecting leaks that cause more than 30% of total flow loss through a system. The work of these authors (1, 2, 21, 22, 23) did not use pressure transducers in locating leak in oil and gas pipelines.

At the present, it is estimated that pipeline systems that are used in many oil-producing countries in the world consist of several hundreds or thousands of kilometers. The economic location of pipe leaks would significantly save running and capital costs in terms of equipment purchase as well as manpower and monitoring costs. A tool for locating leaks, such as proposed in the current study, is necessary and would be economically profitable. Since pipelines are also used for public water systems, the need to pursue the development of locating tools is further strengthened. Saving such costs would make available more funds for maintenance and marketing activities by the organisation, and for the purpose of further product expansion, leading to a better overall performance. Inspired by the above mentioned approaches, it has been established that the knowledge presented in the available literature is still insufficient. Thus, it is necessary to carry out further research and contribute to the elevating of knowledge to a new rank of useful theories and practical ideas. Hence, the desire to enrich leak location and detection literature in oil and gas domain by bridging the gap of sparse model availability on localization, and specifically those that make use of pressure transducer, has motivated this study.

**MATHEMATICAL MODEL**

**Model Assumptions**

The assumptions used in the development of the proposed mathematical model are:

1. The location of the sensors, values and pump in the pipeline are known.
2. There is no free gas present in the pipeline.
3. There are no other reflection zones (like bends, profusion inside the pipeline branch pipes) on the pipeline section, so that the wave reflected by the leak will not be interfered by those reflected by the other reflection points.

4. The pipeline in focus is assumed to be horizontal and frictionless. Most oil pipeline layouts are not inclined, rather, they are horizontal in order to prevent the creation of constant-pressure zones (air cavities).

**Model Development**

The propagation of pressure wave upstream and downstream of the leak is a phenomenon that can be used to locate the leak (especially in long pipelines). The generation of the negative transient informs the liquid upstream the leak to ‘soft pedal’. Consequently, the fluid velocity reduces. All these happen at a wave sonic speed \( C \). The wave speed (celerity) depends on the properties of the fluid and pipe material.

\[
C = \sqrt{\frac{1}{\rho \left( \frac{1}{K} + \frac{d_i}{\left( \frac{d_o - d_i}{2} \right)^2} \right)}}
\]

where \( C \) is in m/hr, \( \rho \) is the density of petroleum, kg/m\(^3\), \( K \) is the bulk modulus of petroleum, N/m\(^2\).

As a result of this, the pressure sensor is de-activated, so also is the controller. Hence, the pump shuts down and generates another negative pressure entrapped between the pump and \( PT \) because pressure transducer constitutes a dead end. Hence, the negative pressure wave retards the flow between the pump and \( PT_i \). The pressure transducer \( PT_2 \) (Figure 1) measures the magnitude and time the wave generated by the pipeline breakage gets to it.

![Figure 1. The pressure transducer](image-url)
On the other hand, the wave generated by the pump shutdown cannot interfere with that propagated upstream of the leak and arriving at the transducer ($PT_2$). The pressure transducer is a total reflection zone. Hence, the wave generated by the pump shutdown exists and propagates between the pump and pressure transmitter (transducer) $PT_1$. Therefore, the positioning of these reflection zones simplifies the analysis. At the downstream and upstream of the leak, the propagation of a negative pressure wave retards the flow. It is assumed that the pressure drop in the downstream side of the leak is insufficient to reduce the fluid pressure to either its vapour pressure or its dissolved gas released pressure, which may be considerably different. This is to prevent the formation of a constant-pressure zone in the fluid flow regime which may cause change in the sign of the negative pressure wave. At the pressure transducers $PT_2$ and $PT_3$, the magnitudes of the pressure recorded just after pressure reduction has occurred in the pipeline should be noted when determining leak area, while the time interval between these pressures, will help in locating the leak. Let the position of $PT_2$ be identified as node 2. Since the pressure transducer is a total reflection zone, it reflects the wave back. The pressure transducer constitutes a dead end zone.

From the wave characteristic equation with $F = +C$

\[
\frac{dp}{dt} \pm \rho \frac{du}{dt} = 0
\]  

[2]

Using finite difference approach,

\[
p_{i}^{t+\Delta t} - p_{i-1}^{t} + \rho c \left( u_{i}^{t+\Delta t} - u_{i-1}^{t} \right) = 0
\]  

[3]

Considering node $i = 2$,

\[
p_{2}^{t+\Delta t} - p_{1}^{t} + \rho c \left( u_{2}^{t+\Delta t} - u_{1}^{t} \right) = 0
\]  

[4]

Adding and subtracting $p_{2}^{t}$ yields,

\[
\left( p_{2}^{t+\Delta t} - p_{2}^{t} \right) + \left( p_{1}^{t} - p_{2}^{t} \right) + \rho c \left( u_{2}^{t+\Delta t} - u_{1}^{t} \right) = 0
\]  

[5]

But, as the velocity at the dead end is constantly zero,

\[
u_{2}^{t+\Delta t} - u_{2}^{t} = 0
\]  

[6]

\[
\rho c \left( u_{2}^{t+\Delta t} - u_{1}^{t} \right) = \left( p_{1}^{t} - p_{2}^{t} \right)
\]  

[7]
Hence, \[ (P_2^{t+\Delta t} - P_2^t) = 2(P_1^t - P_2^t) \] \[ \text{[8]} \]

This implies that the reflection coefficient at the pressure transducer is +1. This means that the wave arriving at the reflection zone, node 2 is reflected as an equal magnitude wave with no change in sign. Now, when the wave (coming upstream to the leak) arrives at the leak (which is assumed to be between \( PT_2 \) and \( PT_4 \) and labelled node 3) part of the wave is reflected and part is transmitted, see Figure 2.

![Figure 2. Transmission and reflection of the wave at the leak site](image)

The wave that is transmitted interferes with that initially propagated downstream at this instance, although a record of the magnitudes propagated in the pipe after pump shutdown are kept by \( PT_2 \) and \( PT_4 \). The pressure transducer \( PT_4 \) at node 4 is also a dead end, and what applies at node 2 it is applicable at node 4, but different pressure wave magnitudes may be recorded at the second and successive times at nodes 2 and 4, due to pressure waves interference and superposition. The pump shutdown after the half-pipe period \( (L/c) \) has simplified the problem because, as earlier mentioned; the shutdown of the pump will generate a negative transient that would be entrapped between the pump and \( PT_1 \). This has really simplified the analysis. If the node \( PT_1 \) were to be a partial reflection point, part of the wave generated by the pump shutdown will interfere with that generated by pipeline rupture. And this would have made the analysis difficult, and even impossible to solve.

Therefore, the time interval between the instants when the first and second pressure waves arrive at nodes 2 and 4 are noted. These time intervals are very essential in determining an approximate location of the leak. Since the behaviour of wave propagation is constant, equation of uniform motion of particles can be used to predict the location of the leak. Making \( PT_2 \) as a point of reference, the location of the leak is:

\[ x_{2-3} = c (\Delta t)_2 \] \[ \text{[9]} \]

where \( x_{2-3} \) is the distance of the leak from the pressure transducer \( PT_2 \).
The location of the leak from the pressure transducer 4 when making reference to node 2 is:

\[ x_{2-3}^1 = L - c (\Delta t)_2 \]  \[10\]

Making reference to node 4, the location of the leak is:

\[ x_{4-3} = c (\Delta t)_4 \]  \[11\]

The location of the leak from \( PT_2 \) when making reference to node 4 is:

\[ x_{4-3}^1 = L - c (\Delta t)_4 \]  \[12\]

Therefore, the approximate location of the leak with respect to \( PT_2 \) is:

\[ (x_{2-3})_{av} = \frac{L + c ((\Delta t)_2 - (\Delta t)_4)}{2} \]  \[13\]

If \((\Delta t)_2 < (\Delta t)_4\), this means that the leak position is at a distance less than half pipe length from \( PT_2 \) and vice versa.

With respect to \( PT_4 \), the approximate location of the leak is:

\[ (x_{4-3})_{av} = \frac{L + c ((\Delta t)_4 - (\Delta t)_2)}{2} \]  \[14\]

When the pump is not in operation and the pipeline ruptures, the operator of the pipeline will not know that it transpired. If in his ignorance he switches on the pump, the pump will shut down no sooner than he switched it on. This is because the pressure in the pipeline is below the set point. The shutdown of the pump will generate a wave which will propagate in the pipeline but entrapped between the pump and \( PT_1 \). In effect, this situation will not support the location of the leak. When the operator has noted this strange phenomenon, he presumes that the pipeline has been ruptured somewhere. Although this approach is impractical to successfully locate the leak, the operator can use Bajoraityte and Bogdevicius’s approach. If, for example, the leak occurred between \( PT_1 \) and the pump, the time intervals between the pressure magnitudes noted after the leak occurrence is relevant in locating the leak. Only, the equation will be:

\[ x_1 = c (\Delta t)_1 \]  \[15\]

where \( x_1 \) is the location of the leak with respect to \( PT_1 \).
Since the negative transient is entrapped between $PT_1$ and the pump, $i_2$ and $i_4$ cannot give us readings to locate the leak. The magnitudes of the pressure waves recorded by the pressure sensing elements are very important in determining the size of the leak in the pipeline. Knowing the reflection and transmission coefficients at the node, the size of the node (leak) can be determined. As mentioned earlier, considering the ‘holidays’ as a branched pipe on the main pipeline, that pipeline is frictionless.

\[ P_3^+ = P_3^- = P_3^L \]
\[ \Delta P_3^+ = \Delta P_3^- = \Delta P_3^L \]

where $P_3^+ = P_C$, $P_3^- = P_A$, $P_3^L = P_B$, $\Delta P_3^+ = \Delta P_C$, $\Delta P_3^- = \Delta P_A$, $\Delta P_3^L = \Delta P_B$

Making reference to the overall transfer function of the control system,
\[
P_C = P^+_3 = \phi^{++}_3 + \phi^{--}_3 \\
\]
\[
P_A = P^-_3 = \phi^{--}_3 + \phi^{--}_3 \\
\]
\[
P_B = P^L_3 = \phi^L_3 + \phi^{--}_3 \\
\]

Also, considering the flow conditions,
\[
Q_A = Q_B + Q_C \\
\]
\[
\Delta Q_A = \Delta A_B + \Delta A_C \\
\]

but,
\[
\Delta Q_A = -\left(\frac{A_A}{\rho c}\right) (\phi^{--}_3 - \phi^{--}_3) \tag{19} \\
\]
\[
\Delta Q_B = -\left(\frac{A_B}{\rho c}\right) (\phi^L_3 - \phi^{--}_3) \tag{20} \\
\]
\[
\Delta Q_C = -\left(\frac{A_C}{\rho c}\right) (\phi^{--}_3 - \phi^{--}_3) \tag{21} \\
\]

From the above conditions,
\[
\phi^{++}_3 = \phi^L_3 = \phi^{--}_3 + \phi^{--}_3 \tag{22} \\
\]

Putting equations [19-22] into [18] yields,
\[
\left(\phi^{--}_3 - \phi^{--}_3\right) = \left(\frac{C}{A_A}\right) \left(\frac{A_B}{C}\right) \varphi^L_3 + \left(\frac{A_C}{C}\right) \varphi^{++}_3 \tag{23} \\
\]

Substituting equation [22] into [23] produces:
\[
\left(\phi^{--}_3 - \phi^{--}_3\right) = \left(\phi^{--}_3 - \phi^{--}_3\right) \left(\frac{A_B}{A_A} + \frac{A_C}{A_A}\right) \tag{24} \\
\]
Let the junction reflection coefficient be defined as:

\[ C_{R,3} = \frac{\phi_3^-}{\phi_3^+} \]  

[25]

Dividing equation [24] by \( \phi_3^+ \) gives

\[ C_{R,3} = \frac{1 - \left( \frac{A_B}{A_A} + \frac{A_C}{A_A} \right)}{1 + \left( \frac{A_B}{A_A} + \frac{A_C}{A_A} \right)} \]  

[26]

Since \( A_B \) represents the area of leak \( j \), hence, let \( A_B = A_3 \). But \( A_A = A_C \), the area of the main pipe.

Let \( A_A = A_C = A \)

Therefore,

\[ C_{R,3} = \frac{\left( \frac{A_3}{A} \right)}{2 + \frac{A_3}{A}} \]  

[27]

Similarly, substituting \( \phi_3^{++} = \phi_3^L \) in equation [23],

\[ \phi_3^{--} - \phi_3^{--} = \phi_3^L \left( \frac{A_B}{A_A} + \frac{A_C}{A_A} \right) \]  

[28]

Adding [22] and [28] together yields,

\[ 2\phi_3^{--} = \phi_3^L \left( \frac{A_3}{A} + 2 \right) \]  

[29]

The junction transmission coefficient,

\[ C_{T,3} = \frac{\phi_3^L}{\phi_3^+} \]  

[30]
Hence,

\[ C_{T,3} = \frac{2}{\left\{ \frac{A_3}{A} + 2 \right\}} \]  

[31]

**Model application**

The pipeline conveying petroleum from two locations in Nigeria, which are at a distance of 3.3 km from each other, has the following installations to supply flow power to the liquid and to detect leak in the pipeline. The control system deactivates the pump when there is leak occurrence in the pipe. Two pressure sensors, which are at a distance of 3 km from each other, record the pressure – time history in the pipeline. The programmable pressure sensor that sends signal to the controller is at a distance of 250 m away from the pump.

**Pipe:** Material – Mild steel, Thickness – 10 mm, Outer diameter - \( \phi 500 \) mm, Elastic modulus – 2.3 \( \times 10^8 \)N/m².

**Induction Motor:** Power rating – 1.5 hp, Armature voltage – 220 V, 50 Hz, Armature current – 10 A, Rotor inertia – 1.13 \( \times 10^{-2} \) Nms²/rad, Inductance – 0.1 H, Armature resistance – 0.45 \( \Omega \), Ratio of torque change to voltage at zero speed – 0.067 Nm/amp, Ratio of torque change to speed change at zero speed and the mid-range voltage – 0.028 Nms/rad.

**Pump:** Delivery pressure – 300 kN/m², Flow rate – 120 m³/min, Type – Centrifugal pump.

**Programmable Logic Controller:** Type – PID (Analog input, Relay output), Voltage – 220 V, Current rating – 2 A, Proportional gain – 3, Integral gain – 15, Derivative gain – 0.3.

**Pressure Transducers:**
- **PT₁:** Type – Programmable pressure transducer, Rating – (-1 to 5 N/m²), ± 10% tolerance, Current – (4-20 mA), Location – 250 m from the source,
- **PT₂:** Type – Non-programmable pressure transmitter, Rating – (-1 to 5 N/m²), ± 10% tolerance, Location – 260 m from the source,
- **PT₄:** Type – Non-programmable pressure transmitter, Rating – (-1 to 5 N/m²), ± 10% tolerance, Location – 3.26 km from the source,

The expected wave speed in the pipeline is

\[ C = \sqrt{\frac{1}{\rho \left( \frac{1}{K} + \frac{1}{tE} \right)}} \]

where the density of petroleum is 873 kg/m³ and bulk modulus of petroleum is 1.65 \( \times 10^9 \) N/m²
The possible magnitude of the pressure transient during the pump start up = \( \rho CV = 20.096 \text{ N/m}^2 \)

The pipeline period = \( \frac{2 \times 3300}{226.42} = 29.155 \)

Assuming the following readings were recorded by the pressure transducers during the transportation of petroleum between the two locations.

### Table 1. Pressure transducers readings

<table>
<thead>
<tr>
<th>Time (h:min:s)</th>
<th>Pressure (N/m²)</th>
<th>Time (h:min:s)</th>
<th>Pressure (N/m²)</th>
<th>Time (h:min:s)</th>
<th>Pressure (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00.00</td>
<td>0.00</td>
<td>8.00.00</td>
<td>0.00</td>
<td>8.00.00</td>
<td>0.00</td>
</tr>
<tr>
<td>8.00.01</td>
<td>53.06</td>
<td>8.00.02</td>
<td>53.00</td>
<td>8.00.14</td>
<td>52.96</td>
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<tr>
<td>8.00.27</td>
<td>42.37</td>
<td>8.00.28</td>
<td>40.06</td>
<td>8.00.42</td>
<td>39.97</td>
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<td>8.00.57</td>
<td>3.78</td>
<td>8.01.10</td>
<td>33.28</td>
</tr>
<tr>
<td>8.01.24-9.55.01</td>
<td>30.00</td>
<td>8.01.25 - 9.55.02</td>
<td>30.00</td>
<td>8.01.39-9.55.15</td>
<td>30.00</td>
</tr>
<tr>
<td>9.55.03</td>
<td>9.904</td>
<td>9.55.05</td>
<td>0.9890</td>
<td>9.55.27</td>
<td>0.009</td>
</tr>
<tr>
<td>9.55.04</td>
<td>9.904</td>
<td>9.55.06</td>
<td>0.0990</td>
<td>9.55.38</td>
<td>0.898</td>
</tr>
<tr>
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<td>0.0099</td>
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</tr>
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<tr>
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</tr>
<tr>
<td>9.55.15</td>
<td>9.904</td>
<td>9.55.28</td>
<td>8.093</td>
<td>9.57.39</td>
<td>0.8093</td>
</tr>
</tbody>
</table>

From the table, it can be deduced that the leak occurred between \( PT_2 \) and \( PT_4 \) because after the sudden drop in pressure to 9.904 N/m² as recorded by \( PT_1 \), this value was recorded for many periods, but the values of the pressures recorded by \( PT_2 \) and \( PT_4 \) fluctuates. Considering \( (\Delta t)_2 = 9.55.05-9.55.03 = 2s \), the corresponding \( (\Delta t)_4 = 9.55.27-9.55.16 = 11s \). Comparing the time difference, it can be seen that the leak location is closer to \( PT_2 \).

Using the transfer function between the motor input and the output speed, \( x_{2-3} = 226.42 \times 2 = 452.84 \text{ m} \). Also, \( 2-3)_{av} = (3000 + 226.42 (2 – 11)/2 = 481.11 \text{ m} \)

The percentage error in the leak location was calculated using Matlab on the leak equation as \( [(481.11-452.84)/452.84] \times 100\% = 6.24\% \).
From Table 1, the coefficient of reflection, $C_{R3} = (-0.989/9.889) = -0.1$

From equation [27], $C_{R,3} = \left( \frac{A_3}{A} \right) \left\{ 2 + \frac{A_3}{A} \right\}$, and pipeline cross-sectional area, $A = 0.18095 \text{m}^2$

\[-0.1 = \left( \frac{A_3}{0.18905} \right) \left\{ 2 + \frac{A_3}{0.18905} \right\} \text{ and } A_3 = 0.0402 \text{ m}^2, \text{i.e. the approximate leak area is } 0.0402 \text{ m}^2\]

As can be seen from Figure 4, there exists a direct relationship between the coefficient of reflection and leak area for the pipes with constant diameters. Thus, the volume of fluid that leaks from a leakage point is affected by the fluid wave reflection. With this knowledge, approximate area of leak in pipes can be estimated. From Figure 5, it is clearly evident that there is a proportional relationship between pipe diameter and leak diameter, under an environment of constant coefficient of reflection. Figure 6 shows a direct relationship between the pipe area and the coefficient of reflection. However, this relationship is steeper at low values of the coefficient of reflection with a less and steady slope as value of coefficient of reflection increases. Combining Figures 4 to 6 will provide a better understanding of leak behaviour in the pipe monitoring.

![Figure 4](image_url)

**Figure 4.** The coefficient of reflection vs. the leak area
Figure 5. The pipe vs. the leak area for a constant coefficient of reflection

Figure 6. The pipe area vs. the coefficient of reflection, at a constant leak area

The area of the leak determined from the analysis shows a reasonable value, which authenticates and justifies the practical application of equation [27] derived in the current work. From the results, as the pipe area increases for a constant value of the coefficient of reflection, the leak area increases since the leak area and pipe area ratio remains constant. Also, as the coefficient of reflection decreases negatively, the leak area increases positively. The relationship between the pipe area and coefficient of reflection is parabolic. For a particular leak area and pipe area, different coefficients of reflection could be obtained.
up to a value of -0.05. The pipe area increases as the coefficient of reflection falls negatively below -0.05 for a constant leak area. The results of these analyses do not deviate from practical values and they justify the claims and assumptions made in the work.

CONCLUSION

A new approach that makes use of pressure transducers for locating leaks in oil and gas pipeline installations was developed. The performance of the proposed approach in leaks location was evaluated using laboratory simulated data for three different pressure transducers that are located at different locations along the length of a pipeline network. The leak location technique was based on a sudden drop in the pressure. The obtained results indicate that the proposed approach can predict leak location in the pipeline with an approximate error of 6.24%. The insight gained from this study indicates also that there is still an insufficient knowledge concerning the modeling of leak area in oil and gas pipeline installations. New hybrids of artificial intelligent techniques such as hybrid particle swarm optimization, hybrid genetic algorithm and hybrid ant cottony optimization have proved successful in fields such as control systems in electrical engineering. These models may be adopted to solve the leak problem in a more precise way.

REFERENCES


МАТЕМАТИЧКИ МОДЕЛ ЛОЦИРАЊА И ОДРЕЂИВАЊА ВЕЛИЧИНЕ ПОВРШИНЕ ПУКОТИНЕ У ЦЕВОВОДИМА

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Хитно лоцирање пукотине и одређивање њене површине у цевоводним инсталацијама нафте и гаса представљају неопходан приступ у спречавању губитака нафтних производа у цевима. Међутим, постоји евидентан недостатак информација у литератури о овом проблему. У овом раду смо моделовали детекцију пукотине у цеви и одређивање њене површине применом две методологије и представили један илустративан пример користећи симулиране податке добијене помоћу програма Matlab. Поређење два два приступа показује да је њихова разлика 6.24, што су-
герише да, уколико се налази ближе мерној станици, утолико ће бити краћи временски интервал између два сукцесивна таласа који пролазе кроз пукотину и стижу до мерне станице. Зависност између површине цеви и коефицијента рефлексије је параболичне природе. Овај прилог је од користи инжењерима који брину о цевоводима како би се спречили економски губици.

Кључне речи: претварач притиска, таласи, детекција, мерење, симулација

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