NEW METHOD FOR DETERMINING COOLING TIME AND PREHEATING TEMPERATURE IN ARC WELDING

by

Valentina M. NEJKOVIĆa, Miroslav S. MILIČEVIĆb*, and Zoran J. RADAKOVIĆc

a Faculty of Electronic Engineering, University of Nis, Nis, Serbia
b High Technical School, University of Belgrade, Belgrade, Serbia
c Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

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Study and research on arc welding have provided identification of errors in formulas for calculating the cooling time \( t_{0.5} \) and other dependent parameters. It is concluded that large errors are present in certain intervals which had caused failures in welding technologies. Incorrect approximation of cooling temperature is replaced by a more accurate approximation used for defining of the new precise algorithm for determining relevant welding parameters.

Key words: cooling time, temperature, welding, preheating, precise method

Introduction

The topic of research of this work is arc welding, for which, according to relevant literature [1-9], a new precise algorithm for calculating the post-weld critical cooling time and cooling rate after welding can be obtained. Also, the new procedure for determining the cooling time and preheating temperature is provided.

Fusion welding processes are of concern, including gas welding, arc welding, and high-energy beam welding. Fusion welding is a joining process that uses the fusion of the base metal to create the weld. Three major types of fusion welding processes are given:

- Gas welding: oxyacetylene welding (OAW).
- Arc welding: shielded metal arc welding (SMAW), gas-tungsten arc welding (GTAW), plasma arc welding (PAW), gas-metal arc welding (GMAW), flux-cored arc welding (FAW), submerged arc welding (SAW), and electroslag welding (ESW).
- High-energy beam welding: electron beam welding (EBW) and laser beam welding (LBW).

Since the electric arc is not involved in the electroslag welding process, it is not exactly an arc welding process. For convenience of discussion, it is grouped with arc welding.

Basic work in this area is studied, researched and presented in [10, 11]. According to our experience and from published references [12-15], a new approach for solving problems of arc welding and the calculation of important parameters has been developed. It is worth to mention that the main result of this research includes a correction of the error found also in the British Standard.

* Corresponding author, e-mail: milicevic.miroslav@mts.rs
Existing theory for calculating the cooling time in arc welding

Before presenting the algorithm for determining specific arc welding parameters, a presentation of existing analytical dependencies is included. They all define links between critical cooling time \( t_{8/5} \), the welding input heat and preheating temperature. The literature [1-11, 16-19] provides 2-D and 3-D models of analytical dependencies. Hence, there is a known relation given by eq. (1) for calculating the cooling time, \( t_{8/5} \), in function of preheating temperature for the adopted 2-D model:

\[
t_{8/5} = \frac{Q^2}{4\pi \lambda \rho c d^2} \left[ \frac{1}{(500 - T_p)^2} - \frac{1}{(800 - T_p)^2} \right]
\]

where \( d \) is the transition thickness; \( Q \) – the heat input, \( T_p \) – the preheat temperature, \( t_{8/5} \) – the cooling time, \( \lambda \) [Js\(^{-1}\)m\(^{-1}\)\(^{-1}\)\(^{\circ}\)C\(^{-1}\)] – the thermal conductivity, \( c \) [Jkg\(^{-1}\)\(^{\circ}\)C\(^{-1}\)] – specific heat, and \( \rho \) [kgm\(^{-3}\)] – the density.

It is known that \( Q = q/v \), where \( q \) for arc welding is \( q = UI\eta \). The \( U \) [V] is welding voltage and \( I \) [A] – the welding current, \( \eta \) –the coefficient of welding efficiency that depends on welding type. The \( Q \) [kJ m\(^{-1}\)] is the heat input for a certain welding rate.

Equation (2) is recognized for the 3-D model in determining welding parameters:

\[
t_{8/5} = \frac{Q}{2\pi \lambda} \left[ \frac{1}{500 - T_p} - \frac{1}{800 - T_p} \right]
\]

Equation (3) is used for the selection of the welding model:

\[
d_{gr} = \left[ \frac{Q}{\rho c \left( \frac{1}{500 - T_p} \right) + \left( \frac{1}{800 - T_p} \right)} \right]^{0.5}
\]

The model of 2-D type is used for defining the thickness for welding if it is equal or less than the value given in eq.(3), otherwise the 3-D model should be used. All these actions are used for improving the accuracy of calculations.

Equations (1) and (2) have been used for a long time while there was no critical estimation in practical applications which included the entire scope of use of variables within different types of arc welding. As an example, in the book [20] the following formulas are provided, and in the Ph. D. thesis [21], eqs.(1) and (2) are used, which implies on the use of these relations until the period of developing this research.

The smaller number of researches has determined the deviation of the calculated cooling time, but there were no corrections and no confirmations of proof for obtaining the correct solution. Therefore, during the development of the British Standard in this area of welding [22], the correction for a more accurate calculation is given by eqs.(4) and (5), respectively, for the 2-D and 3-D model.

\[
t_{8/5} = (4300 - 4.3T_p) Q^2 d^2 \left[ \frac{1}{(500 - T_p)^2} - \frac{1}{(800 - T_p)^2} \right] F_2
\]

\[
t_{8/5} = (6700 - 5T_p) Q \left[ \frac{1}{500 - T_p} - \frac{1}{800 - T_p} \right] F_3
\]
Authors of this work used applied researches of numerical examinations and concluded that eqs.(4) and (5) have significant deviations in some parts of the scope of use. The work in [23] has developed a new model 2.5D for calculating the cooling time:

\[ t_{8/5}^{2.5D} = \gamma t_{8/5}^{3D} + \beta t_{8/5}^{2D} \]  

(6)

It is used for calculation of newly introduced coefficients through use of results from the 2-D and 3-D models and experimental results. Such calculations offer correct results for one scope of variables, because the author has not confirmed and proved it theoretically.

According to the researched problems, authors of this work applied a relation for obtaining the correct results for all arc welded material thicknesses.

**Development of relations and forming of algorithm for precise calculation**

Development of analytical dependency for cooling temperature is conducted according to experience from [24] and from [10, 16-19]. For example, for two steel plates that need to be welded, the temperature distribution can be described by the heat equation:

\[ \frac{\partial^2 T}{\partial t} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]  

(7)

where the thermal conductance is given:

\[ a = \frac{\lambda}{c\gamma} \]  

(8)

If the temperature \( T \) is equated by thickness of plates and not dependent on the \( z \)-axis, the welding is conducted along the \( x \)-axis. The source of heat is an element of volume \( dx \) \( dy \) and height \( d \). According to [10, 16], the solution of eq. (7) is presented in the form of:

\[ T(r, x) = \frac{q}{2\pi\lambda d} \exp\left( -\frac{vx}{2a} \right) K_0 \left( r \left( \frac{v^2}{4ad} + \frac{b}{a} \right)^{0.5} \right) \]  

(9)

where \( K_0 \) is modified Bessel function of 2nd kind and 0 class. Since the case of arc welding includes higher values of \( q \), eq. (9) can be transformed into the form:

\[ T(y_0, t) = \frac{q}{vd\sqrt{4\pi\lambda c\gamma t}} \exp\left( -\frac{y^2}{4at} - bt \right) \]  

(10)

Equation (10) develops in the case when the highest cooling rate is in the welding zone. Hence, it can be confirmed that \( y = 0 \) and \( \exp(-bt) = 1 \):

\[ T(t) = \frac{q}{vd\sqrt{4\pi\lambda c\gamma t}} \]  

(11)

Cooling time for arc welding, \( t_{8/5} \), can be defined as a time difference between \( t_5 \) and \( t_8 \). In this case, \( t_8 \) is a time when \( T(t) \) reaches 800 °C by cooling, and \( t_5 \) is a time when it reaches 500 °C:

\[ t_{8/5} = t_5 - t_8 \]  

(12)
When \( t_{8/5} \) is less than real for welded steel, preheating at temperature \( T_p \) should be conducted. The times \( t_8 \) and \( t_5 \) can be calculated if eq. (11) is squared and solved for \( t \):

\[
t = \frac{q^2}{\nu^2 d^2 4\pi \lambda c \gamma \left(T - T_p\right)^2} \tag{13}
\]

Replacement in eq. (13), \( T = 800 \, ^\circ C \) will provide \( t_8 \), or \( T = 500 \, ^\circ C \) will provide \( t_5 \) in eqs. (14) and (15):

\[
t_8 = \frac{q^2}{\nu^2 d^2 4\pi \lambda c \gamma \left(800 - T_p\right)^2} \tag{14}
\]

\[
t_5 = \frac{q^2}{\nu^2 d^2 4\pi \lambda c \gamma \left(500 - T_p\right)^2} \tag{15}
\]

If the replacement is placed for eqs. (14) and (15) in eq. (12), we arrive at eq. (1), presenting the 2-D Rosenthal model for calculation of time \( t_{8/5} \). However, authors of this work already concluded that this relation does not provide accurate values in many applications.

Therefore, in case when \( y = 0 \), the eq.(10) becomes:

\[
T(t) = \frac{q}{\nu d \sqrt{4\pi \lambda c \gamma t}} \exp(-bt) \tag{16}
\]

The following example is provided for determining the deviation of eq. (11) relating to eq. (16).

Example 1. There is heat in the amount of \( q_1 = 15333 \, J/cm \) for arc welding, where steel plates of thickness 7.4 mm are welded with preheating to \( T_p = 180 \, ^\circ C \). There should be \( T(t) \) graphics drawn and time of cooling \( t_{8/5} \) should be provided in case of application of eqs. (11) and (16). The value \( q_1 \) is derived from expression \( q_1 = q/v \), so after replacement of values for constants, the relation eq. (11) becomes:

\[
T(t) = \frac{4523}{\sqrt{t}} \tag{17}
\]

and eq. (16) becomes:

\[
T(t) = 4523 \frac{e^{-bt}}{\sqrt{t}} \tag{18}
\]

The value \( b \) is a function of the applied steel for welding and its geometry which can be confirmed in [25-28]. Figure 1 shows the diagram for cooling temperatures in this example as a function of time, \( t \). It can be seen that function \( T_1(t) \) has a slower decreasing rate and its cooling time is equal to \( t_{8/5} = 50 \) seconds. The temperature \( T_2(t) \) has a faster decreasing rate and cooling time value \( t_{8/5} = 15 \) seconds. If there is only one example of welding, it can be concluded that the curve \( T_1(t) \) for higher values of time, \( t \), can have larger deviations regarding the curve \( T_2(t) \). Therefore, it is concluded that the 2-D Rosenthal model of the equation for calculating \( t_{8/5} \) [25, 28] develops large deviations. A confirmation of this lies in a series of papers [26, 27] which have emphasized that this equation demands large values for cooling time, \( t_{8/5} \), which can be considered as a paradox.
Figure 1 presents relative error, $\varepsilon$, in function of time, $t$, where increase of $t$ increases the error. Therefore, the approximation using eq. (17) is possible only for small values of $t$. The relative error of deviation $T_1(t)$ relating to $T_2(t)$ can be presented by expression:

$$
\varepsilon = \frac{T_1(t) - T_2(t)}{T_2(t)} \times 100 = (e^{bt} - 1) \times 100 \% \quad (19)
$$

According to relation (19), for $t = 0$, the error, $\varepsilon$, has the value equal to zero and it is also considered as small for small values of $t$ which correspond to even smaller values of inserted heat, $q$. Due to increasing of time, $t$, and to the time, $T(t)$, decreases to 500 °C, the error of deviation increases, which shows that relation (18) should be used for temperature, $T_2(t)$. Figure 1 shows that there are two plots for cooling temperature, after welding, where $T_1(t)$ corresponded to eqs. (1)-(5) for calculation of 2-D and 3-D of the Rosenthal model. Since $T_1(t)$ significantly deviates from the accurate value $T_2(t)$, it shows the reason why there is a large deviation during the determination of $t_{8/5}$ in full scope of time, $t$. The plot for $\varepsilon$ presents a relative percentage deviation which describes the dependency, eq. (19).

**Iterative calculation and correction of formulas in the British Standard**

Problematic formulas are used for decades by experts and researchers in the area of heat transfer and as such, they are also incorporated into the British Standard. Even improved versions had produced significant deviations in numerical calculations.

It is proven that eq. (16) should be used for calculation. It is transcendent regarding variable $t$. Also, it is important to determine $t_5$ and $t_8$, but it is impossible to provide this explicitly. Authors of this work introduced the Newton iterative method into the calculations. For the function $f(t) = 0$, it is:

$$
t_{i+1} = t_i - \frac{f(t_i)}{f'(t_i)}, \quad i = 0, 1, 2, \ldots \quad (20)
$$

where $i$ is the number of iterations, $f(t_i)$ – the function from which $t_i$ is calculated, and $f'(t_i)$ – the first derivative of the function. The process of iterations $i = 0$ is started by initial selection of expected value. Due to good convergence of the iterative process and lack of good initial solution, the procedure examines the solution for $t$ within 2 to 3 iterations. One example is given for illustrating the application of the precise algorithm.

**Example 2.** Arc welding includes heat input $q_1 = 16911$ J/cm, the sheet thickness is $d = 7.4$ mm and the preheating temperature $T_p = 180$ °C. This example is taken from [26] in order to compare results.
After replacement for known constants, the expression is:

$$T(t) = 4988.5 \frac{e^{-bt}}{\sqrt{t}}$$  \hspace{1cm} (21)

For calculating $t_8$ the valid expression is:

$$T(t) = 800 - 180 = 620 \, ^\circ\text{C}$$  \hspace{1cm} (22)

and for $t_5$, the eq. (21) can be used:

$$T(t) = 500 - 180 = 320 \, ^\circ\text{C}$$  \hspace{1cm} (23)

There is:

$$f(t) = 4990e^{-bt} - 620\sqrt{t} = 0$$  \hspace{1cm} (24)

or

$$f(t) = e^{-bt} - 0.124248\sqrt{t} = 0$$  \hspace{1cm} (25)

and the first derivative has the form

$$f'(t) = -be^{-bt} - \frac{0.062124}{\sqrt{t}}$$  \hspace{1cm} (26)

The solution is obtained from 3 iterations, while the starting iteration provided $t_{8,0} = 15$ seconds:

$$t_{8,1} = 28.71 \, \text{s}$$
$$t_{8,2} = 30.30 \, \text{s}$$
$$t_{8,3} = 30.34 \, \text{s}$$  \hspace{1cm} (27)

The similar for $t_5$ can be obtained, where $t_{5,0} = 25$ seconds, as shown in eq. (26):

$$t_{5,1} = 51.41 \, \text{s}$$
$$t_{5,2} = 57.39 \, \text{s}$$
$$t_{5,3} = 57.60 \, \text{s}$$  \hspace{1cm} (28)

Finally, the cooling time $t_{8/5}$ is provided in the form:

$$t_{8/5} = 57.6 - 30.34 = 27.26 \, \text{s}$$  \hspace{1cm} (29)

Through the dilatometric method, described in detail in [7] and presented through works of authors [26, 27], the value of 23.5 seconds is obtained. The solution derived from the precise method of the author of this work is valid, since solutions provided through the dilatometric method according to [7] depend on chemical-mechanical properties of the steel, and the pre-process of steel production. These are two different ways for defining the same parameter, the cooling time $t_{8/5}$.

In order to obtain values for $t_8$ and $t_5$, in eqs. (14) and (15) using the derivation from eq. (11), it can be provided through eq. (16), which further provides the expressions:

$$t_8 = \frac{q^2}{v^2d^2} \frac{e^{-2bq}}{4\pi\lambda\gamma (800 - T_p)^2}$$  \hspace{1cm} (30)
and

\[ t_s = \frac{q^2}{v^2d^24\pi\lambda c\gamma (500 - T_p)^2} e^{-2bt_s} \] (31)

Finally, the solution for \( t_{8/5} \):

\[ t_{8/5} = \frac{q^2}{4\pi\lambda p c (vd)^2} \left[ e^{-2bt_{8/5}} - \frac{e^{-2bt_s}}{(500 - T_p)^2} - \frac{e^{-2bt_s}}{(800 - T_p)^2} \right] \] (32)

which presents the new relation for determining \( t_{8/5} \), from which the precise and exact relation of all variables in the welding process can be defined.

However, it is much easier to apply a special iterative process for determining the special values for \( t_8 \) and \( t_5 \), and to define \( t_{8/5} \) by simple substitution. The reason for this is the fact that eq. (32) is much more difficult for solving in practice.

Results and discussion

Accuracy of the obtained results by using the method in this work shall be tested first by using finite element method (FEM) simulation and calculation via two examples. The first example is related to welding of thin steel plates and the second is related to a thicker steel. All the examples include 2-D and 3-D methods of the Rosenthal model.

Example 3. Arc welding of steel sheets of thickness 7.4 mm, input heat \( q_1 = 11058 \text{ J/cm} \) and preheating temperature of 180 °C. Calculation of the cooling time \( t_{8/5} \) by using the 2-D theoretical model has improved the formulas in the British Standard, the new methods from this work and the FEM method, successfully are applied in papers [26, 27]. Also, the experimental data are obtained by dilatometric method for determining the cooling time \( t_{8/5} \). After the calculation, the data are presented in tab. 1.

Table 1. Comparative values of cooling time \( t_{8/5} (s = 7.4 \text{ mm}) \) with preheat temperature, including FEM results

<table>
<thead>
<tr>
<th>Input heat ([\text{kJ/cm}^2])</th>
<th>Preheat temperature ([\text{°C}])</th>
<th>Equation 2-D or experiment</th>
<th>Experiment</th>
<th>FEM</th>
<th>Equation in this paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.1</td>
<td>180</td>
<td>84.9</td>
<td>27</td>
<td>31</td>
<td>31.6</td>
</tr>
</tbody>
</table>

Table 1 presents the largest deviation by 2-D Rosenthal model corrected for British Standard in eq. (4). This result significantly deviates from the experimental result. Application of the FEM method provides a result 31 seconds, which is near to the experimental result of 27 seconds. The result obtained by using the algorithm in this work is 31.6 seconds, which is nearly the same as the result from the FEM method.

Example 4. The steel sheet of thickness 29 mm is welded [26, 27] with input heat of 16.5 kJ/cm and preheating temperature of 204 °C. Calculations are made as in Example 3. The calculated data are given in tab. 2.

Table 2 shows that \( t_{8/5} \) has the value of 12 seconds, the FEM analysis provides a result of 11.5 seconds and the algorithm from this work provides a result of 11.67 seconds. Since the results of FEM analysis and presented in this work are nearly the same, it proves the justifi-
cation that the algorithm here presented really displays a new precise way for calculating the cooling time \( t_{8/5} \).

Since the analysis of results from previous two examples in a view of statistical reasoning is small, there are larger researches found through the results of numerous works [26, 27, 29-35]. Also, all these results are compared to results obtained in this work. Results are reviewed in tab. 3, from where it is obvious that a larger deviation is provided through eq. (1). Some bit smaller deviations are given by eq. (4) which presents the formula from the British Standard. There are results provided through experiments [26, 27]. However, the results obtained in this work [31] are not presented, since this work proves that there is no quoted formula. In the work [28], results are included for cooling time \( t_{8/5} \). These results have a larger error.

Table 2. Comparative values of cooling time \( t_{8/5} \) \((s = 29 \text{ mm})\) with preheat temperature, including FEM results

<table>
<thead>
<tr>
<th>Input heat ([\text{kJcm}^{-1}])</th>
<th>Preheat temperature ([\text{°C}])</th>
<th>Equation 3-D or experiment</th>
<th>FEM/new precise equation</th>
<th>FEM/new precise equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.5</td>
<td>204</td>
<td>14.9</td>
<td>12</td>
<td>11.5</td>
</tr>
</tbody>
</table>

As final and major results, data obtained using the new precise algorithm from this work are imported. The justification of implementing these methods is largely confirmed due to a fact that they are nearly the same as results from the FEM analysis.

Nowadays, theory and practice use five formulas. Some of these formulas are very hard for solving since their form is not appropriate and they are complex. The algorithm shown here uses only one formula and it is very simple for calculating \( t_{8/5} \). These features provide results appropriate to many researchers and practitioners, due to short time of calculation.

The calculation of \( t_{8/5} \) is presented here since numerous papers have applied this parameter, and also the eqs. (1)-(5). Results used in this paper make it possible to calculate also the preheating temperature when \( t_{8/5} \) is defined for particular types of steel. This can be done by formulating eq. (16) as the difference between \( T(t_{8}) \) and \( T(t_{8} + t_{8/5}) \), whose numerical value is 300 °C, and \( t_{8} \) is calculated by a subsequent application of the iterative method, eq.(20). The preheating temperature \( T_{p} \) is then determined by substituting this value into eq. (30).

**Conclusion**

Research in this work is conducted in a way to include a long period of arc welding, starting from the first theoretical solutions, professional and scientific works, books, Ph. D.
papers and different standards. According to results achieved through the research, including the results from heat transfer, there are assumptions for identifying the deviations and errors in this area of study.

It is concluded that certain relations produce large deviations, so the authors had found and proved the correct formulas, using mathematical and numerical analyses, which helped them to form a new and precise method of calculating the specific parameters. Also, there are errors identified in the British Standard, and they have also been removed by deriving the exact formulas without any errors or deviations.

Nowadays, the theory uses more of these formulas, so unsolved problems and solutions with significant deviations are included in practice. Problems in these theories have been solved by the authors of this work by introducing only a single formula and a simple way of calculation which could be favourable for further work and research.

Apart from identifying poor approximations in eqs. (1)-(5), an exact relation has been derived, eq. (32). The procedures for calculating the critical cooling time, \( t_{8/5} \), or the preheating temperature, \( T_p \), are defined in this work, depending on the requirements in solving welding parameters. Here, the arc welding procedure is particularly considered, due to the bulk number of papers that deal with this technology and also because of the involved British Standard, that also deals with arc welding, and in the favour of a suggested correction. It is also true that this procedure may be applied to other types of welding technology.

**Nomenclature**

- \( c \) – specific heat, [\( \text{Jkg}^{-1}\text{°C}^{-1} \)]
- \( d \) – transition thickness, [mm]
- \( I \) – welding current, [A]
- \( Q \) – heat input, [\( \text{kJmm}^{-1} \)]
- \( t_{8/5} \) – cooling time, [s]
- \( T_p \) – preheating temperature, [°C]
- \( U \) – welding voltage, [V]
- \( v \) – welding speed, [\( \text{mms}^{-1} \)]

**Greek letters**

- \( \eta \) – heat efficiency
- \( \lambda \) – thermal conductivity, [\( \text{Jkg}^{-1}\text{°C}^{-1} \)]
- \( \rho \) – density, [\( \text{kgm}^{-3} \)]

**References**


